Fluctuations of particle ratios and the abundance of hadronic resonances

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In this letter we will argue that the event-by-event fluctuations of the ratio of positively over negatively charged pions provides a measurements of the number of rho and omega mesons right after hadronization. This finding can be utilized to put the hypothesis of chemical equilibration in relativistic heavy ion collisions to a test.

The question to which extent the matter created in relativistic heavy ion collisions is equilibrated is central to the interpretation of many observables for the existence of a new phase of matter. A detailed analysis of the inclusive single particle yields of several hadronic species has led many authors [1–4] to conjecture that rather early in the collision chemical equilibrium has been reached. Indeed assuming chemical equilibrium at an early stage of the collisions a rather impressive agreement with a large body of data can be obtained by adjusting just a few parameters, namely the temperature, the baryon chemical potential and the strangeness suppression factor (for details see e.g. [2]). However, this analysis has to rely on the abundance of final state 'stable' particles and thus has to infer the number of most hadronic resonances present inside the system. Some information about the abundance of unstable resonances, such as the ρ and ω meson, may be obtained through the observation of the electromagnetic decay into dileptons. However, this only provides a time integrated yield and thus gives only limited information about the abundance of these resonances right after hadronization and/or chemical freeze-out.

In this letter we propose to study the event-by-event fluctuations of particle ratios, in particular the ratio π^+/π^- in order to put a strong constraint on the relative abundance of some unstable resonance right after hadronization/chemical freeze-out. We will show that the fluctuations of π^+/π^- are quite sensitive to the number of the *primordial* ρ_0 and ω mesons. In more general terms, the investigation of the event-by-event fluctuations of particle ratios provides a crucial test of the hypothetical chemical equilibration – to see if it also predicts two particle correlations correctly in addition to the single particle inclusive data.

The key point to our argument, that the fluctuation of the π^+/π^- is indeed sensitive to the particle numbers at chemical and *not* at thermal freeze-out, is the observation [5,6] that the pion number does not change during the course of the evolution of the system through the hadronic phase. Typical relaxation times for pion number changing processes are of the order of 100 fm/c, much longer than the lifetime of the system, which is about 10 fm/c. In addition, as we shall argue in more detail below, charge exchange processes, which in principle could affect the π^+/π^--fluctuations, lead only to small corrections. Finally, considering fluctuations of the multiplicity ratio eliminates the effect of volume fluctuations which are present even with the tightest centrality selection.

Let us now define some notations. We define the fluctuation δN_i by

$$N_i = \langle N_i \rangle + \delta N_i \tag{1}$$

where $\langle N_i \rangle$ is the average number of the particle species i. Then the variance is given by

$$\Delta(N_i, N_i) \equiv \langle \delta N_i \delta N_i \rangle = \left\langle {N_i}^2 \right\rangle - \langle N_i \rangle^2 = w_i \langle N_i \rangle \tag{2}$$

Here we introduced the notation

$$\Delta(N_i, N_j) \equiv \langle N_i\, N_j \rangle - \langle N_i \rangle \langle N_j \rangle = \langle \delta N_i \delta N_j \rangle \tag{3}$$

and defined w_i to be the ratio $\Delta(N_i, N_i)/\langle N_i \rangle$.

The absence of correlations make the fluctuation in the multiplicity very close to the (inclusive) average number of particles. Hence, in a classical thermal system, $w_i = 1$, since there are no correlations among the particles. Bose-Einstein or Fermi-Dirac statistics introduce correlations so that

$$w_i = 1 \pm \left\langle n_i^2 \right\rangle / \langle n_i \rangle \tag{4}$$

with

$$\langle n \rangle = \int \frac{d^3p}{(2\pi)^3} n_{\pm}(p); \qquad \left\langle n^2 \right\rangle = \int \frac{d^3p}{(2\pi)^3} n_{\pm}^2(p) \tag{5}$$

where, $(+)$ stands for Bosons, and $(-)$ stands for Fermions [7]. For the systems of interest here, however, the corrections due to quantum statistics are small; for a pion gas at a temperature of 170 MeV, $w_\pi = 1.13$ [8].

In general however, w_i will *not* be equal to $1 \pm \left\langle n_i^2 \right\rangle / \langle n_i \rangle$ due to additional correlations introduced by

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interactions and resonances. This will be discussed in detail below.

Given the above notation, the fluctuation of the ratio is given by [12]

$$\frac{\Delta(R_{12},R_{12})}{\langle R_{12}\rangle^2} = \left(\frac{\Delta(N_1,N_1)}{\langle N_1\rangle^2} + \frac{\Delta(N_2,N_2)}{\langle N_2\rangle^2} - 2\,\frac{\Delta(N_1,N_2)}{\langle N_1\rangle\langle N_2\rangle}\right) \tag{6}$$

The last term in Eq. (6) takes into account correlations between the particles of type 1 and type 2. This term will be important if both particle types originate from the decay of one and the same resonance. For example, in case of the π^+/π^-ratio, the ρ_0, ω etc. contribute to these correlations. Also, volume fluctuations contribute here.

(i) Volume fluctuations: Even though data are often selected according to some centrality trigger, the impact parameter and thus the volume of the created system, still fluctuates considerably. Assuming, that the particle abundance scales linearly with the volume, volume fluctuations translate directly into fluctuations of the particle number.

However, by considering ratios of particles these fluctuations cancel to leading order. This can be seen as follows. Note that we can rewrite Eq. (6) as

$$\frac{\Delta(R_{12},R_{12})}{\langle R_{12}\rangle^2} = \left\langle\left(\frac{\delta N_1}{\langle N_1\rangle} - \frac{\delta N_2}{\langle N_2\rangle}\right)^2\right\rangle \tag{7}$$

Assuming that the volume fluctuation separates from the density fluctuation, we write

$$N = n_{\rm ave}(\langle V\rangle + \delta V)(1 + \delta n/n_{\rm ave}) \tag{8}$$

where $n_{\rm ave} = \langle N\rangle / \langle V\rangle$. Then to the first order,

$$\frac{\delta N}{\langle N\rangle} = \frac{\delta V}{\langle V\rangle} + \frac{\delta n}{n_{\rm ave}} = \frac{\delta V}{\langle V\rangle} + \frac{\delta_n N}{\langle N\rangle} \tag{9}$$

where $\delta_n N = \langle V\rangle\,\delta n$ is the number fluctuation due to the density fluctuation. Clearly, the volume fluctuation part will be cancelled in Eq. (7) and hence $\Delta(N_i,N_j)$ in Eq. (6) can be simply replaced with

$$\Delta_n(N_i,N_j) = \langle\delta_n N_i\,\delta_n N_j\rangle \tag{10}$$

From now on, unless otherwise signified, we will omit the subscript n from $\Delta_n(N_i,N_j)$.

Let us now turn to the discussion of the density fluctuations. In the physical system we consider, the density fluctuation is mainly due to the thermal fluctuation. As already mentioned above, in absence of any resonances/interactions the thermal fluctuations $\Delta(N_1,N_1)$ are simply given by Eq. (2), with w_1 slightly different from unity due to quantum statistics. Furthermore in a thermal system the correlation term vanishes, i.e. $\Delta(N_i,N_j) = \delta_{ij}\,\Delta(N_i,N_j)$ since $\langle N_1\,N_2\rangle = \langle N_1\rangle\langle N_2\rangle$ in that case. This changes, however, once interactions, in particular resonances, are present in the system.

(ii) Effect of resonances: A fundamental assumption of the statistical model is that at the chemical freeze-out time, all the particles including resonances are in thermal and chemical equilibrium. The expansion afterwards breaks the equilibrium. However, as discussed above the total number of π^+ and π^- given by

$$\langle N_i\rangle = \langle N_i\rangle_T + \sum_R \langle R\rangle_T\,\langle n_i\rangle_R \tag{11}$$

remains constant from this time on. Here the subscript T on $\langle N_i\rangle_T$ and $\langle R\rangle_T$ denotes the average number of particles and resonances at the freeze-out time and $\langle n_i\rangle_R$ is the average number of the particle type i produced by the decay of a single resonance R.

The presence of resonances which decay into the particles of interest affects the fluctuations of each individual particle ($\Delta(N_i,N_i)$) [9]. Resonances decaying into both particle species of interest also affect the correlation term ($\Delta(N_i,N_j)$). A single resonance contribution to $\Delta(N_i,N_j)$ is given by [11]

$$\frac{\Delta_R(N_i,N_j)}{\langle R\rangle} = \langle n_i n_j\rangle_R + (w_R - 1)\,\langle n_i\rangle_R\,\langle n_j\rangle_R \tag{12}$$

where we defined

$$\langle n_i n_j\rangle_R \equiv \sum_{r\in\text{branches}} b_r^R (n_i^R)_r (n_j^R)_r \tag{13}$$

and

$$\langle n_i\rangle_R \equiv \sum_{r\in\text{branches}} b_r^R (n_i^R)_r \tag{14}$$

Here the index r runs over all branches, b_r^R is the branching ratio of the r-th branch, and $(n_i^R)_r$ represents the number of i-particles produced in that decay mode. At the chemical freeze-out, the system is in equilibrium and hence the number of different particle species are uncorrelated. In the final state where all the the resonances have decayed, the correlation is given by

$$\Delta(N_i,N_j) = w_i^T\,\langle N_i\rangle_T\,\delta_{ij} + \sum_R \langle R\rangle_T\left(\langle n_i n_j\rangle_R + (w_R^T - 1)\,\langle n_i\rangle_R\,\langle n_j\rangle_R\right) \tag{15}$$

where $w_i^T\,\langle N_i\rangle_T$ denotes the part of the variance due only to the statistical fluctuations at the chemical freeze-out time.

Putting everything together we get for the fluctuations of the ratio

$$\frac{\Delta(R_{12},R_{12})}{\langle R_{12}\rangle^2} = \frac{1}{\langle N_2\rangle}\left(D_{11} + D_{22} - 2D_{12}\right) \tag{16}$$

with

$$D_{11} = \frac{\langle N_2 \rangle}{\langle N_1 \rangle} F_1 \tag{17}$$

$$D_{22} = F_2 \tag{18}$$

$$D_{12} = \sum_R \langle n_1 n_2 \rangle_R \frac{\langle R \rangle_T}{\langle N_1 \rangle} \tag{19}$$

where

$$F_i = \left(w_i^T r_i + \sum_R \langle n_i^2 \rangle_R \frac{\langle R \rangle_T}{\langle N_i \rangle} \right)$$
$$= \left(1 + (w_i^T - 1) r_1 + \sum_R \left(\langle n_i^2 \rangle_R - \langle n_i \rangle_R \right) \frac{\langle R \rangle_T}{\langle N_i \rangle} \right) \tag{20}$$

and we defined $r_i \equiv \langle N_i \rangle_T / \langle N_i \rangle$. Here we regarded $(w_R^T - 1)$ to be negligibly small. For a typical resonance of $m_R \sim 1\,\mathrm{GeV}$, $(w_R^T - 1) < 10^{-2}$ assuming the temperature of 170 MeV. (In the numerical results presented below, these terms, though small, are included.). Note that we have factored out $1/\langle N_2 \rangle$ to separate out the explicit dependence on the system size.

Before we turn to the practical applications, a few comments are in order at this point. First, the effect of the correlations introduced by the resonances should be most visible when $\langle N_1 \rangle \simeq \langle N_2 \rangle$ since the branching fraction n_i^R should enter with about the same weight. In this case, a resonance decaying always into a pair of particles "1" and "2" contributes about equally to D_{11}, D_{22} and D_{12} and hence contributes negligibly to $(D_{11}+D_{22} - 2D_{12})$. On the other hand, the presence of such resonances does influence the total number of particle "2", $\langle N_2 \rangle$. Hence, for instance, ρ^0 and ω will always *reduce* the fluctuation in π^+/π^- compared to the statistical fluctuation. Second, when $\langle N_2 \rangle \gg \langle N_1 \rangle$, as in the K to π ratio, the fluctuation is dominated by the less abundant particle type and the resonances feeding into it.

After this general formulation of the problem, let us now turn to the calculation of the negative to positive pion ratio. The formalism developed above can be easily applied to the case of π^+/π^-fluctuations by setting

$$N_1 = \pi^+, \quad N_2 = \pi^-$$

Typically, $\langle \pi^+ \rangle / \langle \pi^- \rangle \simeq 1$. In table (I) we show the most important contributions from hadronic resonances. This calculation includes mesons and baryons up to $m \sim 1.5$ GeV as listed in the particle data book. Weakly decaying strange particles are regarded as stable, but letting them decay changes the main result very little. The values of temperature, baryon chemical potential and the strangeness chemical potential are the same as those in [2], i.e., $T = 170$ MeV, $\mu_b = 270$ MeV and $\mu_s = 74$ MeV.

As already pointed out, the ρ^0 and ω contribute about 50 % of the correlations. Furthermore, the correlation-term $2D_{+-}$ is *seventy percent* of the individual contributions D_{++} or D_{--}. This is a sizable correction which should be visible in experiment. Furthermore, since only very few resonances have decay channels with more than one charged pion of the the same charge, $\langle n_\pm^2 \rangle_R - \langle n_\pm \rangle_R \simeq 0$ to a good approximation. Also, $(w_\pm^T - 1) r_\pm \simeq 4$ % at $T = 170$ MeV. Hence, the fluctuations of the individual pion contributions are very close to the statistical limit of $D_{++} \simeq D_{--} \simeq 1$. Thus the resonances contribute predominantly to the correlation term D_{+-}.

TABLE I. Contributions from different hadrons to the fluctuations of the π^+/π^- ratio. Contributions are given in fraction of the total result.

Particle	$\frac{D_{++}}{D_{++}^{tot}}$	$\frac{D_{--}}{D_{--}^{tot}}$	$\frac{D_{+-}}{D_{+-}^{tot}}$	$\frac{n_+}{n_+^{tot}}$	$\frac{n_-}{n_-^{tot}}$
π^+	0.32	0.00	0.00	0.31	0.00
π^-	0.00	0.32	0.00	0.00	0.31
η	0.02	0.02	0.06	0.02	0.02
ρ^+	0.08	0.00	0.00	0.09	0.00
ρ^0	0.08	0.08	0.24	0.09	0.09
ρ^-	0.00	0.08	0.00	0.00	0.09
ω	0.07	0.07	0.21	0.08	0.08
Others	0.44	0.44	0.55	0.43	0.44

TABLE II. Total values

	D_{++}^{tot}	D_{--}^{tot}	$2D_{+-}^{tot}$	n_+^{tot}	n_-^{tot}
Values	1.09	1.09	0.76	0.20 fm^{-3}	0.20 fm^{-3}

Using $\langle \pi^+ \rangle = \langle \pi^- \rangle = \langle \pi^+ + \pi^- \rangle /2$ and $\langle \pi^+ + \pi^- \rangle = 220$ from the recent NA49 results [10] on event-by-event fluctuations of the transverse momentum and the K/π-ratio, we obtain for the total fluctuation π^+/π^- ratio

$$\frac{\Delta(R_{+-}, R_{+-})}{\langle R_{+-} \rangle^2} \equiv \sigma_{+-}^2 = 0.0128 \tag{21}$$

so that $\sigma_{+-} = 0.113$ A more useful quantity, to consider, however, is the ratio of the above total fluctuations over the purely statistical value. The latter can be possibly obtained in experiment by the analysis of mixed events. Possible artificial contributions due to experimental uncertainties, such as particle identification etc. should cancel to a large extent in this ratio, and thus a comparison with theory becomes more meaningful. The value for the statistical fluctuation for us is simply given by

$$\left(\sigma_{+-}^{stat.} \right)^2 = \frac{1}{\langle \pi^+ \rangle} + \frac{1}{\langle \pi^- \rangle} \tag{22}$$

since in mixed events all correlations, even the quantum statistical ones, are absent, i.e. $w_+ = w_- = 1$. Our result for this ratio is

$$\frac{\sigma_{+-}^2}{(\sigma_{+-}^{stat})^2} = 0.70 \tag{23}$$

Note that this ratio is independent of $\langle \pi^- \rangle$. Thus the correlations introduced by the presence of resonances *reduce* the fluctuations by 30 %. Looking at table (I), the most important contribution of the correlation come from the ρ and ω meson. Or in other words, the measurement of the fluctuations of the π^+/π^- provides a strong constraint on the initial number of ρ^0 and ω-mesons.

Doing the same analysis for the K/π ratio, where preliminary data exist [10], we find

$$\frac{\sigma_{K/\pi}^2}{(\sigma_{K/\pi}^{stat})^2} = 1.04 \tag{24}$$

in good agreement with the data. Note, however that our value for the fluctuation itself, $\sigma_{K/\pi} = 0.17$ differs from the experimental value of $\sigma_{K/\pi}^{exp} = 0.23$ indicating the effect of additional fluctuations from particle identifications etc. [13]. The reason (see [11] for details), is that in case of the K/π ratio, the fluctuation is dominated by the contribution from the kaon, which is largest due to the small abundance.

Let us close by discussing some possible caveats. First, there is the question of acceptance cuts. Clearly, one should *not* consider the particle ratio of the full 4π acceptance. In this case charge conservation will impose severe constraints and reduce the fluctuations of the π^+/π^- ratio. However, as long as a limited acceptance in, say, rapidity is considered, the constraint from charge conservation is minimal and our assumption of a grand-canonical ensemble are well justified. Limited acceptance on the other hand may reduce the effect of resonances on the correlation term, as some of the decay products may end up outside the acceptance. In order to estimate this effect we have performed a Monte-Carlo study. Given the rapidity distribution of charged particles for a Pb+Pb collision at SPS-energies [10], a rapidity window of $\Delta y = 1$ changes the above results by less then one percent. On the other hand this window covers only a fraction of the observed rapidity distribution, so that constraints from charge conservation are negligible. Finally there are the charge exchange reactions such as $\pi^+ + \pi^- \leftrightarrow \pi^0 + \pi^0$. These reactions in principle could change the π^+/π^--ratio in a given event. However, detailed balance requires the net change to be close to zero. In addition, for a given event, these reactions influence *not* the difference between π^+ and π^- but only their sum. Thus, again we expect only small corrections to the above result since the sum is large compared to the expected changes.

The reaction with baryons, notably $\pi^- + p \leftrightarrow \pi^0 + n$ might be more effective. But again, detailed balance and the fact that the nucleon/pion ratios is so small should make these corrections insignificant. A detailed quantitative investigation of these corrections will be presented in [11].

In conclusion, we propose to study the event-by-event fluctuations of the π^+/π^--ratio in relativistic heavy ion collisions. This measurement will provide important information about the abundance of short lived resonances right after hadronization and/or chemical freeze-out. It will further impose a strong test on the validity of the chemical equilibration hypothesis in these reactions. If chemical equilibrium is reached with the values for temperature and chemical potential extracted from the single particle distributions, we predict that the fluctuations of the π^+/π^--ratio should be about 70 % of the statistical ones. It would be also of great interest to study these fluctuations in proton-proton and peripheral heavy ion collisions, where the particle abundances also seem to indicate chemical equilibrium.

In addition, any value of the $\sigma_{+-}/\sigma_{+-}^{stat}$ significantly larger than 1 cannot be explained with a simple hadronic gas picture and thus would indicate new physics. One possible scenario might be the Quark Gluon Plasma bubble formation [12]. Thus the π^+/π^- ratio as a function of E_T may serve as an alternative signal for the QGP.

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